



Oxford Cambridge and RSA

Monday 05 October 2020 – Afternoon

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

Time allowed: 2 hours 40 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **144**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

Section A (36 marks)Answer **all** the questions.

- 1** Using standard summation of series formulae, determine the sum of the first n terms of the series $(1 \times 2 \times 4) + (2 \times 3 \times 5) + (3 \times 4 \times 6) + \dots$,
where n is a positive integer. Give your answer in fully factorised form. [6]

- 2 (a)** The matrices $\mathbf{M} = \begin{pmatrix} 0 & 1 & a \\ 1 & b & 0 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} b & -5 \\ -1 & c \\ -1 & 1 \end{pmatrix}$ are such that $\mathbf{MN} = \mathbf{I}$.

Find a , b and c . [5]

- (b)** State with a reason whether or not \mathbf{N} is the inverse of \mathbf{M} . [1]

- 3 In this question you must show detailed reasoning.**

Find $\int_0^{\frac{1}{3}} \frac{1}{\sqrt{4-9x^2}} dx$, expressing your answer in terms of π . [4]

- 4** The roots of the equation $2x^3 - 5x + 7 = 0$ are α , β and γ .

(a) Find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [4]

(b) Find an equation with integer coefficients whose roots are $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$. [4]

- 5 Fig. 5 shows the curve with polar equation $r = a(3 + 2 \cos \theta)$ for $-\pi \leq \theta \leq \pi$, where a is a constant.

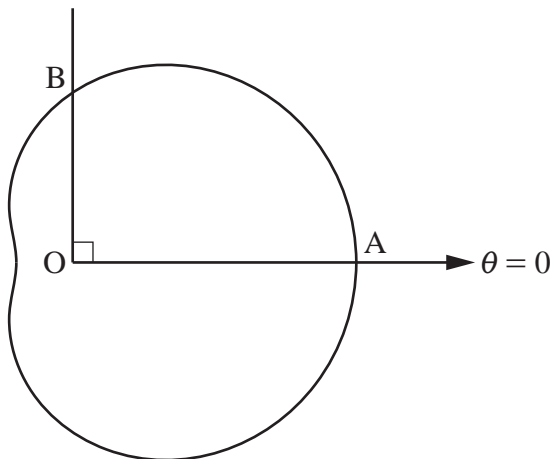


Fig. 5

- (a) Write down the polar coordinates of the points A and B. [2]
- (b) Explain why the curve is symmetrical about the initial line. [2]
- (c) **In this question you must show detailed reasoning.**
- Find in terms of a the exact area of the region enclosed by the curve. [4]

- 6 The complex number z satisfies the equation $z^2 - 4iz^* + 11 = 0$.

Given that $\text{Re}(z) > 0$, find z in the form $a + bi$, where a and b are real numbers. [4]

4

Section B (108 marks)

Answer **all** the questions.

- 7 Prove by mathematical induction that $\sum_{r=1}^n (r \times r!) = (n+1)! - 1$ for all positive integers n . [6]

- 8 (a) Given that the lines $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 2 \\ k \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ meet, determine k . [5]

(b) **In this question you must show detailed reasoning.**

Find the acute angle between the two lines. [4]

- 9 A linear transformation of the plane is represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & -2 \\ \lambda & 3 \end{pmatrix}$, where λ is a constant.

(a) Find the set of values of λ for which the linear transformation has no invariant lines through the origin. [5]

(b) Given that the transformation multiplies areas by 5 and reverses orientation, find the invariant lines. [3]

10 **In this question you must show detailed reasoning.**

The region in the first quadrant bounded by curve $y = \cosh \frac{1}{2}x^2$, the y -axis, and the line $y = 2$ is rotated through 360° about the y -axis.

Find the exact volume of revolution generated, expressing your answer in a form involving a logarithm. [7]

11 In this question you must show detailed reasoning.

In Fig. 11, the points A, B, C, D, E and F represent the complex sixth roots of 64 on an Argand diagram. The midpoints of AB, BC, CD, DE, EF and FA are G, H, I, J, K and L respectively.

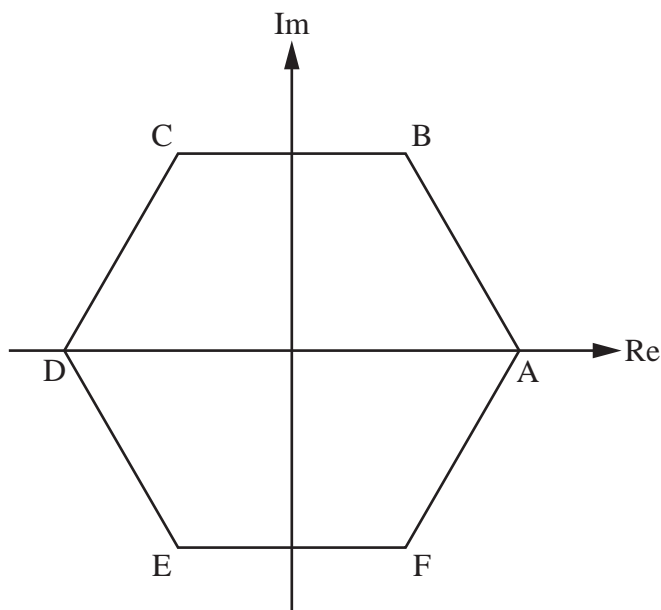


Fig. 11

- (a) Write down, in exponential ($re^{i\theta}$) form, the complex numbers represented by the points A, B, C, D, E and F. [2]

- (b) When these complex numbers are multiplied by the complex number w , the resulting complex numbers are represented by the points G, H, I, J, K and L.

Find w in exponential form. [4]

- (c) You are given that G, H, I, J, K and L represent roots of the equation $z^6 = p$.

Find p . [2]

- 12 (a)** Given that $z = \cos \theta + i \sin \theta$, express $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ in simplified trigonometric form. [2]

- (b) By considering $\left(z + \frac{1}{z}\right)^3 \left(z - \frac{1}{z}\right)^3$, find constants A and B such that

$$\sin^3 \theta \cos^3 \theta = A \sin 6\theta + B \sin 2\theta. \quad [6]$$

- 13 (a)** Using exponentials, prove that $\sinh 2x = 2 \cosh x \sinh x$. [2]
- (b)** Hence show that if $f(x) = \sinh^2 x$, then $f''(x) = 2 \cosh 2x$. [2]
- (c)** Explain why the coefficients of odd powers in the Maclaurin series for $\sinh^2 x$ are all zero. [2]
- (d)** Find the coefficient of x^n in this series when n is a positive even number. [3]

- 14** Solve the simultaneous differential equations

$$\frac{dx}{dt} + 2x = 4y, \quad \frac{dy}{dt} + 3x = 5y,$$

given that when $t = 0$, $x = 0$ and $y = 1$. [11]

- 15 (a)** Show that the three planes with equations

$$x + \lambda y + 3z = -12$$

$$2x + y + 5z = -11$$

$$x - 2y + 2z = -9$$

where λ is a constant, meet at a unique point except for one value of λ which is to be determined. [3]

- (b)** In the case $\lambda = -2$, use matrices to find the point of intersection P of the planes, showing your method clearly. [3]

The line l has equation $\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+2}{-2}$.

- (c)** Find a vector equation of l . [2]
- (d)** Find the shortest distance between the point P and l . [4]
- (e) (i)** Show that l is parallel to the plane $x - 2y + 2z = -9$. [3]
- (ii)** Find the distance between l and the plane $x - 2y + 2z = -9$. [2]

16 The population density P , in suitable units, of a certain bacterium at time t hours is to be modelled by a differential equation. Initially, the population density is zero, and its long-term value is A .

(a) One simple model is to assume that the rate of change of population density is directly proportional to $A - P$.

(i) Formulate a differential equation for this model. [1]

(ii) Verify that $P = A(1 - e^{-kt})$, where k is a positive constant, satisfies

- this differential equation,
- the initial condition,
- the long-term condition. [3]

An alternative model uses the differential equation

$$\frac{dP}{dt} - \frac{P}{t(1+t^2)} = Q(t),$$

where $Q(t)$ is a function of t .

(b) Find the integrating factor for this differential equation, showing that it can be written in the form $\frac{\sqrt{1+t^2}}{t}$. [8]

(c) Suppose that $Q(t) = 0$.

(i) Show that $P = \frac{At}{\sqrt{1+t^2}}$. [4]

(ii) Find the time predicted by this model for the population density to reach half its long-term value. Give your answer correct to the nearest minute. [2]

(d) Now suppose that $Q(t) = \frac{te^{-t}}{\sqrt{1+t^2}}$.

Show that $P = \frac{At - te^{-t}}{\sqrt{1+t^2}}$. [You may assume that $\lim_{t \rightarrow \infty} te^{-t} = 0$.] [5]

It is found that the long-term value of P is 10, and P reaches half this value after 37 minutes.

(e) Determine which of the models proposed in parts (c) and (d) is more consistent with these data. [2]

END OF QUESTION PAPER

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